

Problem 9) Drawing the diameter BB' divides each of the angles \widehat{ABC} and \widehat{AOC} into two angles. On the left-hand-side of the circle we now have $\beta = \widehat{ABO}$ and $\gamma = \widehat{AOB'}$, for which we are going to prove that $\beta = \gamma/2$. The same line of reasoning then applies to the remaining angles on the right-hand-side, namely, \widehat{OBC} and $\widehat{B'OC}$.

The angle γ is the external angle of the AOB triangle which is supplementary to \widehat{AOB} (that is, they add up to 180°). Similarly, $\alpha + \beta$ is supplementary to \widehat{AOB} . Therefore, $\alpha + \beta = \gamma$. However, the triangle AOB is isosceles, because $AO = BO = R$. Therefore, $\alpha = \beta$. Consequently, $\beta = \gamma/2$, which completes the proof.

